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Evaluating the Impact of the linear Distance on Biochemical Oxygen demand and the Concentration of dissolved Oxygen of a River for fixed initial Conditions using a Computational Approach

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Abstract— *This paper has examined the impact of the linear distance on the concentration of biochemical oxygen demand (BOD) and the concentration of dissolved oxygen (DO) of a river for fixed initial conditions using the method of a numerical scheme called ODE45. Our results are presented and discussed quantitatively.*
Keywords— ***Linear distance, biochemical oxygen demand, dissolved oxygen, initial conditions and numerical scheme***

I. INTRODUCTION

The primary aim of this study is to evaluate the effect of varying the linear distance on the concentration biochemical oxygen demand (L) and the concentration of dissolved oxygen(C) of a river for fixed initial conditions. Since mathematical formulations that describe the growth of L and C depend on several factors such as the linear distance, average flow velocity, deoxygenation coefficient (k_1) and reoxygenation or reaeration coefficient (k_2) [4]

In this present analysis, we are interested to measure numerically the effect of varying the linear distance on L and C with fixed initial conditions. Several other researchers have studied other aspect of modelling the growth of L and C using different independent variables. See Tadeusz et.al [5], Borsuk and Stow [6],Tyagi*et.al* [1],Kaushik *et. al* [2],Runnel [3],Adrian and Sanders [7-8].

II. MATHEMATICAL FORMULATIONS

Under some simplifying assumptions and following Bank [4], we have considered the steady flow of a river with an average velocity, u_0 defined by the following system of first order ordinary differential equations:

$$u_0 \frac{dL}{ds} = -k_1 L \qquad (1)$$

$$u_0 \frac{dC}{ds} = -k_1 L - k_2 D \qquad (2)$$

Here, s represents the linear distance along the river, C represents the concentration of dissolved oxygen (mg/l), L represents the concentration of the biochemical oxygen demand, D represents the difference between the equilibrium concentration of oxygen(mg/l), C_s and the concentration of the dissolved oxygen C, k_1 represents deoxygenation coefficient and k_2 represents the reoxygenation or reaeration coefficient.

For the purpose of this analysis, we have considered the following parameter values; k_1 = 0.25/day,k_2 = 0.5/day,u_0 = 25km/day and s=20km.

III. METHOD OF ANALYSIS

We have adopted a computational approach in our investigation using MATLAB function, ODE45 being more computationally efficient than ODE23, ODE23TB and ODE15s.

IV. RESULTS AND DISCUSSION

The full results of implementing the method above are presented and discussed here as follows

In Table 1, apart from the initial condition data having the effect of zero, as the linear distance ranges from 0.1 to 2.0, the biochemical oxygen demand(BOD) data increases monotonically from 16.801 to 16.833 approximately whereas the new biochemical oxygen demand data when the linear distance is 2km ranges from 16.0001 to 16.8033. On the basis of these calculations, a small value of the linear distance has dominantly predicted a relatively small increase in the original BOD data. For the same value of the linear distance, the concentration of dissolved oxygen (DO) dominantly tend to increase from the percentage effect of 1.797 to 42.673 approximately.

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Therefore, when the linear distance (s) is decreased from its original value of 20km to a smaller value of 2km, the BOD suffers some sort of depletion which mimics biodiversity loss whereas the concentration of DO also suffers some sort of depletion which depicts biodiversity gain.

Table.1: Numerical evaluation of s = 2 km on L(s) and C(s) using ODE45 MATLAB numerical scheme

s (km)	L(s)	$L_m(s)$	Effect (%)	C(s)	C_m	Effect (%)
0.0	16.800000000000001	16.800000000000001	0	8.199999999999999	8.199999999999999	0
0.1	16.801680084002800	16.800168000840003	0.008999595012149	8.039292516003346	8.183784222023517	1.797318678634219
0.2	16.803360336022401	16.800336003360023	0.017998380097184	7.881767419306669	8.167600844871377	3.626514338199693
0.3	16.805040756075606	16.800504007560075	0.026996355328024	7.727361695757802	8.151449803806715	5.488135857309895
0.4	16.806721344179216	16.800672013440177	0.035993520777533	7.576013575901285	8.135331034221988	7.382740972110313
0.5	16.808402100350044	16.800840021000351	0.044989876518575	7.427662516854911	8.119244471638769	9.310896304382643
0.6	16.810083024604889	16.801008030240606	0.053985422624026	7.282249171178213	8.103190051707426	11.273177575114346
0.7	16.811764116960568	16.801176041160961	0.062980159166792	7.139715369909354	8.087167710206932	13.270169624557560
0.8	16.813445377433887	16.801344053761433	0.071974086219684	7.000004092654296	8.071177383044539	15.302466630188349
0.9	16.815126806041658	16.801512068042040	0.080967203855553	6.863059451288826	8.055219006255589	17.370672124119313
1.0	16.816808402800699	16.801680084002800	0.089959512147264	6.728826661220535	8.039292516003195	19.475399215365673
1.1	16.818490167727823	16.801848101643724	0.098951011167658	6.597252025729908	8.023397848578046	21.617270604276364
1.2	16.820172100839851	16.802016120964836	0.107941700989544	6.468282908359138	8.007534940398093	23.796918809004765
1.3	16.821854202153599	16.802184141966148	0.116931581685764	6.341867717867196	7.991703728008354	26.014986176596054
1.4	16.823536471685888	16.802352164647679	0.125920653329115	6.217955881701312	7.975904148080599	28.272125113539559
1.5	16.825218909453543	16.802520189009446	0.134908915992427	6.096497831541957	7.960136137413159	30.568998093120658
1.6	16.826901515473388	16.802688215051464	0.143896369748509	5.977444977814520	7.944399632930612	32.906277889909610
1.7	16.828584289762247	16.802856242773753	0.152883014670135	5.860749695801085	7.928694571683590	35.284647582954733
1.8	16.830267232336951	16.803024272176327	0.161868850830127	5.746365301151517	7.913020890848467	37.704800794038839
1.9	16.831950343214324	16.803192303259202	0.170853878301247	5.634246036539803	7.897378527727172	40.167441686256943
2.0	16.833633622411202	16.803360336022397	0.179838097156293	5.524347048442684	7.881767419746875	42.673285197908584

As presented in Table 2, when the linear distance is 4km, the DO concentration dominantly suffers a depletion value of 4.125 percent whereas the DOB suffers a depletion level of 14.786 percent.

Table.2: Numerical evaluation of s = 4 km on L(s) and C(s) using ODE45 MATLAB numerical scheme

s (km)	L(s)	$L_m(s)$	Effect (%)	C(s)	C_m	Effect (%)
0.0	16.800000000000001	16.800000000000001	0	8.199999999999999	8.199999999999999	0
0.2	16.968842807014028	16.833633622411206	0.796808516293956	8.204797486268953	8.200831979764779	0.048331558588877
0.4	17.139382512477251	16.867334579379353	1.587267994630792	8.211180045814109	8.201727838369431	0.115113873912631
0.6	17.311636170419384	16.901103005708290	2.371429024210758	8.219133177461869	8.202687455343781	0.200090712280754
0.8	17.485621006460736	16.934939036471803	3.149341792238669	8.228642835959603	8.203710710967631	0.302991945196807
1.0	17.661354419117814	16.968842807014024	3.921056084770991	8.239695426605207	8.204797486268904	0.423534348413190
1.2	17.838853981991541	17.002814452950116	4.686621292407089	8.252277796003900	8.205947663020558	0.561422362753927
1.4	18.018137445071769	17.036854110166672	5.446086410965256	8.266377227069119	8.207161123738798	0.716348912028986
1.6	18.199222736969713	17.070961914822401	6.199500047083739	8.281981430185079	8.208437751680004	0.887996177304051
1.8	18.382127966248802	17.105138003348536	6.946910418885843	8.299078538429528	8.209777430838964	1.076036419911552
2.0	18.566871423702221	17.139382512449508	7.688365361492188	8.317657099127032	8.211180045945831	1.280132757484997
2.2	18.753471583710652	17.173695579103363	8.423912327675321	8.337706069285407	8.212645482464374	1.499939980874743
2.4	18.941947106565809	17.208077340562451	9.153598393284245	8.359214807525310	8.214173626588952	1.735105321205366
2.6	19.132316839855623	17.242527934353827	9.877470257888831	8.382173069722500	8.215764365242787	1.985269250533639
2.8	19.324599820834703	17.277047498279948	10.595574250118222	8.406571001299886	8.217417586074969	2.250066230281866

3.0	19.518815277837508	17.311636170419082	11.307956328294944	8.432399133068092	8.219133177458740	2.529125486636630
3.2	19.714982632696707	17.346294089125998	12.014662085688633	8.459648373866683	8.220911028488521	2.822071731913389
3.4	19.913121502184882	17.381021393032380	12.715736751140083	8.488310006595810	8.222751028978214	3.128525906938418
3.6	20.113251700481744	17.415818221047527	13.411225194231569	8.518375681193870	8.224653069458269	3.448105868165186
3.8	20.315393240644969	17.450684712358761	14.101171925900946	8.549837410853304	8.226617041173981	3.780427090566907
4.0	20.519566337094034	17.485621006432122	14.785621103392078	8.582687565409842	8.228642836082626	4.125103315587242

As presented in Table3, both BOD and DO data corresponding to the scenario when the linear distance is 19 km are both vulnerable to the ecological risk of depletion.

Table.3: *Numerical evaluation of s=19 km on L(s) and C(s) using ODE45 MATLAB numerical scheme*

s (km)	**L(s)**	**L_m(s)**	**Effect (%)**	**C(s)**	**C_m**	**Effect (%)**
0.00	16.800000000000001	16.800000000000001	0	8.199999999999999	8.199999999999999	0
0.95	16.968842807014028	16.960360506362399	0.049987502083071	8.204797486268953	8.204519843952129	0.003383902129073
1.90	17.139382512477251	17.122251696793942	0.099950016698902	8.211180045814109	8.210470878472739	0.008636606887358
2.85	17.311636170419384	17.285688182020223	0.149887556229367	8.219133177461869	8.217840641028593	0.015725945855460
3.80	17.485621006460736	17.450684712380884	0.199800133303496	8.228642835959603	8.226617041078542	0.024618821371214
4.75	17.661354419117814	17.617256178838044	0.249687760254869	8.239695426605207	8.236788355901000	0.035281288369238
5.70	17.838853981991541	17.785417614667171	0.299550449699926	8.252277796003900	8.248343223419743	0.047678624998082
6.65	18.018137445071769	17.955184196463602	0.349388213959845	8.266377227069119	8.261270638310128	0.061775414050413
7.60	18.199222736969713	18.126571245865804	0.399201065638510	8.281981430185079	8.275559945124220	0.077535612884305
8.60	18.382127966248802	18.299594230581235	0.448989017044743	8.299078538429528	8.291200834561140	0.094922633059924
9.50	18.566871423702221	18.474268766142647	0.498752080769838	8.317657099127032	8.308183336880980	0.113899408609275
10.50	18.753471583710652	18.650610616953628	0.548490269110335	8.337706069285407	8.326497818333518	0.134428472996639
11.40	18.941947106565809	18.828635698078592	0.598203594644919	8.359214807525310	8.346134974850783	0.156472024893450
12.35	19.132316839855623	19.008360076308371	0.647892069657918	8.382173069722500	8.367085828628602	0.179992001696971
13.30	19.324599820834703	19.189799971984563	0.697555706715369	8.406571001299886	8.389341722088039	0.204950142087457
14.25	19.518815277837508	19.372971760085530	0.747194518089289	8.432399133068092	8.412894314604476	0.231308055463453
15.2	19.714982632696707	19.557891972085763	0.796808516333225	8.459648373866683	8.437735576728544	0.259027280682622
16.15	19.913121502184882	19.744577297062708	0.846397713706926	8.488310006595810	8.463857787057609	0.288069350897890
17.1	20.113251700481744	19.933044583591787	0.895962122751337	8.518375681193870	8.491253526707192	0.318395848008390
18.1	20.315393240644969	20.123310840874471	0.945501755713984	8.549837410853304	8.519915676319934	0.349968462504169
19.0	20.519566337094034	20.315393240644145	0.995016625087231	8.582687565409842	8.549837410847832	0.382749043486130

In the scenario when the value of the linear distance is 24km, the BOD data and DO data both predict dis-similar outcomes of biodiversity gain at the magnitudes of 4.08 and 1.69 approximately.

Table.4: Numerical evaluation of s=24km on L(s) and C(s) using ODE45 MATLAB numerical scheme

S (km)	**L(s)**	**L_m(s)**	**Effect (%)**	**C(s)**	**C_m**	**Effect (%)**
0.0	16.800000000000001	16.800000000000001	0	8.199999999999999	8.199999999999999	0
1.2	16.968842807014028	17.002814452950119	0.200200133400075	8.204797486268953	8.205947663020744	0.014018344190903
2.4	17.139382512477251	17.208077340631380	0.400801067973822	8.211180045814109	8.214173626259942	0.036457371889687
3.6	17.311636170419384	17.415818221048418	0.601803605409934	8.219133177461869	8.224653069467660	0.067159052987820
4.8	17.485621006460736	17.626067009522615	0.803208550671353	8.228642835959603	8.237362111292683	0.105962495965639
6.0	17.661354419117814	17.838853981963872	1.005016708423678	8.239695426605207	8.252277796136362	0.152704303735884
7.2	17.838853981991541	18.054209780346234	1.207228886856160	8.252277796003900	8.269378071432803	0.207218853407753
8.4	18.018137445071769	18.272165415991083	1.409845893859552	8.266377227069119	8.288641775534900	0.269338645626682
9.6	18.199222736969713	18.492752275175757	1.612868540862311	8.281981430185079	8.310048616189203	0.338894577834092

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10.8	18.382127966248802	18.716002122496484	1.816297638993181	8.299078538429528	8.333579159067543	0.415716280768486
12.0	18.566871423702221	18.941947106613256	2.020134002932861	8.317657099127032	8.359214807383747	0.499632381588278
13.2	18.753471583710652	19.170619763694447	2.224378447061137	8.337706069285407	8.386937791042495	0.590470824324796
14.4	18.941947106565809	19.402053023301217	2.429031789323921	8.359214807525310	8.416731147347591	0.688059119745343
15.6	19.132316839855623	19.636280211915818	2.634094847365054	8.382173069722500	8.448578710741080	0.792224647072071
16.8	19.324599820834703	19.873335058968912	2.839568442408802	8.406571001299886	8.482465094555206	0.902794887993985
18.0	19.518815277837508	20.113251700453599	3.045453395376097	8.432399133068092	8.518375681316117	1.019597707500153
19.2	19.714982632696707	20.356064685099138	3.251750530782704	8.459648373866683	8.556296605493994	1.142461569985276
20.4	19.913121502184882	20.601808978072750	3.458460672839792	8.488310006595810	8.596214744346906	1.271215797576297
21.6	20.113251700481744	20.850519967303310	3.665584649367792	8.518375681193870	8.638117701625815	1.405690766800771
22.8	20.315393240644969	21.102233467273070	3.873123287881386	8.549837410853304	8.681993798935343	1.545718143298003
24.0	20.519566337094034	21.356985725409423	4.081077419270573	8.582687565409842	8.727832060552656	1.691131059317352

In Table 5, a similar observation has been made when the linear distance is 28km

Table.5: Numerical evaluation of s=28km on L(s) and C(s) using ODE45 MATLAB numerical scheme

s (km)	L(s)	L_m(s)	Effect (%)	C(s)	C_m	Effect (%)
0.0	16.800000000000001	16.800000000000001	0	8.199999999999999	8.199999999999999	0
1.4	16.968842807014028	17.036854110166708	0.400801067734369	8.204797486268953	8.207161123739166	0.028807992813573
2.8	17.139382512477251	17.277047498428676	0.803208551131918	8.211180045814109	8.217417585361281	0.075963984620597
4.2	17.311636170419384	17.520627243015095	1.207228886619149	8.219133177461869	8.230730346875211	0.141099665414157
5.6	17.485621006460736	17.767641086930293	1.612868541330936	8.228642835959603	8.247062097777855	0.223843253200373
7.0	17.661354419117814	18.018137445075471	2.020134002698293	8.239695426605207	8.266377227104956	0.323820227791383
8.4	17.838853981991541	18.272165416150241	2.429031789800695	8.252277796003900	8.288641774922507	0.440653839067506
9.8	18.018137445071769	18.529774789830057	2.839568442176743	8.266377227069119	8.313823406834006	0.573965819144084
11.2	18.199222736969713	18.791016059005720	3.251750531267716	8.281981430185079	8.341891368442390	0.723376872579773
12.6	18.382127966248802	19.055940427164582	3.665584649138332	8.299078538429528	8.372816461443886	0.888507352628465
14.0	18.566871423702221	19.324599820977639	4.081077420012247	8.317657099127032	8.406571000908022	1.068977726796661
15.4	18.753471583710652	19.597046897890323	4.498235488902247	8.337706069285407	8.443128792877289	1.264409211788364
16.8	18.941947106565809	19.873335059066982	4.917065533238274	8.359214807525310	8.482465094321364	1.474424208899361
18.2	19.132316839855623	20.153518457197254	5.337574251406441	8.382173069722500	8.524556592166359	1.698646893347600
19.6	19.324599820834703	20.437652009808097	5.759768374470364	8.406571001299886	8.569381365787484	1.936703615093749
21.0	19.518815277837508	20.725791407291833	6.183654654618187	8.432399133068092	8.616918867395984	2.188223439332804
22.4	19.714982632696707	21.017993126596192	6.609239876978035	8.459648373866683	8.667149886942150	2.452838509416955
23.8	19.913121502184882	21.314314439480295	7.036530847972133	8.488310006595810	8.720056533792125	2.730184536335689
25.2	20.113251700481744	21.614813426593457	7.465534407227392	8.518375681193870	8.775622203920426	3.019901121460067
26.6	20.315393240644969	21.919548985965601	7.896257415835795	8.549837410853304	8.833831562812666	3.321632193833946
28.0	20.519566337094034	22.228580847294552	8.328706767603888	8.582687565409842	8.894670515231434	3.635026295014554

Our present cutting-edge contribution to knowledge compliments the earlier formulations and analysis provided by Bank [4] and also has moved the frontier of knowledge forward to evaluate the effect of varying the linear distance on the BOD and DO data which were not previously considered by Bank [4].

V. CONCLUSION

We have successfully utilised a computationally efficient numerical scheme known as the Runge-Kutta MATLAB ODE 45 function to predict a biodiversity loss and a biodiversity gain on the BOD and DO data due to a variation of the linear distance along a river.

In our future investigation we will be interested to study the impact of varying the k_1 and k_2 on the BOD and DO dependent variables. Our present predictions have shown the response of BOD data and DO data when the linear distance increases. An alternative mathematical model of interaction between BOD and DO can be constructed to tackle this challenging problem which we did not consider in this pioneering paper.

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